Characterizing Bad Semidefinite Programs: Normal Forms and Short Proofs

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Talk at the Balas Memorial Conference

Carnegie Mellon University, October 2019

A Semidefinite Program (SDP)

$$\begin{aligned} &\sup_x \; c^T x \\ &\quad s.t. \; \sum_{i=1}^m x_i A_i \preceq B. \end{aligned} \qquad (SDP)$$

Here

- A_i, B are symmetric matrices, $c, x \in \mathbb{R}^m$.
- $A \preceq B$ means that $B - A$ is symmetric positive semidefinite (psd).
- An $n \times n$ matrix Y is positive semidefinite, if all principal subdeterminants are nonnegative.
- Equivalently, if $v^T Y v \geq 0 \, \forall v \in \mathbb{R}^n$.

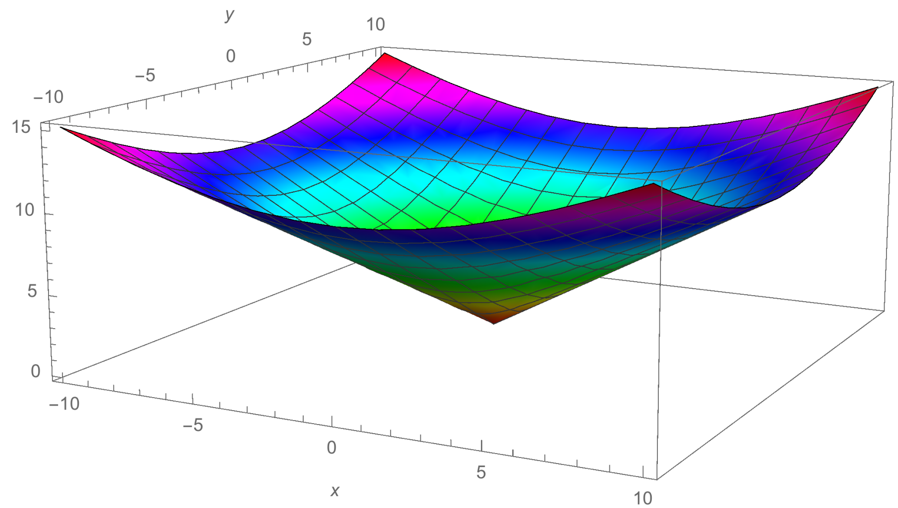
Why is SDP important: applications in

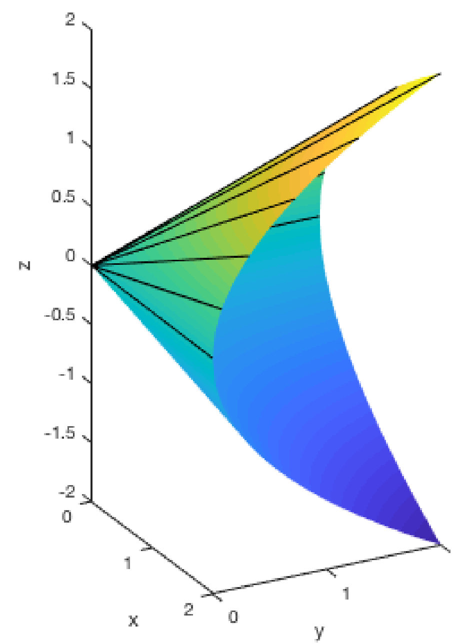
- 0–1 Integer programming.
- Approx algorithms
- Chemical engineering
- Chemistry
- Coding theory
- Control theory
- Combinatorial opt
- Discrete geometry
- Eigenvalue optimization
- Facility planning
- Finance
- Geometric optimization
- Global optimization
- Graph visualization
- Inventory theory
- Machine learning
- Matrix analysis
- PDEs
- Probability theory
- Robust optimization
- Signal processing
- Statistics
- Structural optimization

Why is SDP important: beautiful theory in

- Duality
- Interior point methods
- Geometry

Some nice pictures of SDP feasible sets





. . . and many interesting glitches

- **Interior point methods do not work as well as in LP.**
- **Nor does duality theory.**

SDP duality

The primal-dual pair of SDPs:

$$\sup_x \; c^T x \qquad\qquad \inf_Y \; B \bullet Y$$

$$s.t. \; \sum_{i=1}^m x_i A_i \preceq B \qquad\qquad Y \succeq 0$$

$$A_i \bullet Y = c_i \; (i = 1, \ldots, m).$$

Easy: If x, Y are feasible $\Rightarrow \; c^T x \leq B \bullet Y$.

Ideally: $\exists x^*, Y^*$ such that $c^T x^* = B \bullet Y^*$.

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Easy: If x, Y are feasible $\Rightarrow$ $c^T x \leq B \bullet Y$.

Ideally: $\exists x^*, Y^*$ such that $c^T x^* = B \bullet Y^*$.

But: SDPs, unlike LPs can be pathological: nonattainment, positive gaps.

Pathological SDPs often defeat SDP solvers.

Ben-Tal, Nemirovsky: “Is there something wrong with SDP duality?”

Pathology # 1: nonattainment in dual

Primal:

$$\begin{array}{ll} \sup & 2x_1 \\ s.t. & x_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \preceq \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{array} \quad \Leftrightarrow \quad \begin{array}{ll} \sup & 2x_1 \\ s.t. & \begin{pmatrix} 1 & -x_1 \\ -x_1 & 0 \end{pmatrix} \succeq 0 \end{array}$$

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Dual: Dual variable is $Y \succeq 0.$

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$$\inf \ y_{11}$$

$$s.t. \begin{pmatrix} y_{11} & 1 \\ 1 & y_{22} \end{pmatrix} \succeq 0$$

Unattained $\inf = 0 :$ $y_{11} > 0$ **is feasible, but** $y_{11} = 0$ **is not.**

Same story in pictures

Primal:

$$\sup \ 2x_1$$

$$s.t. \ -x_1^2 \geq 0$$

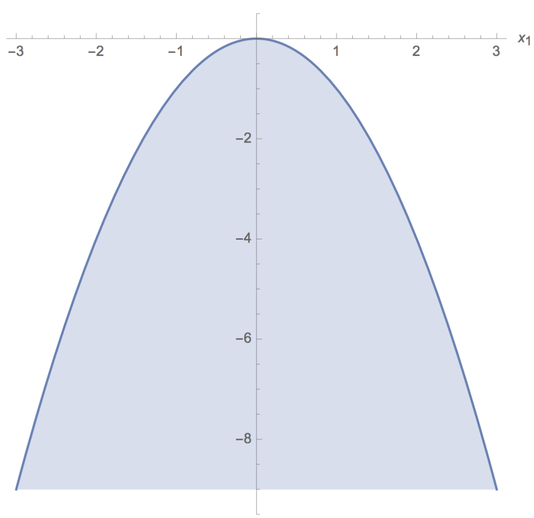
Dual:

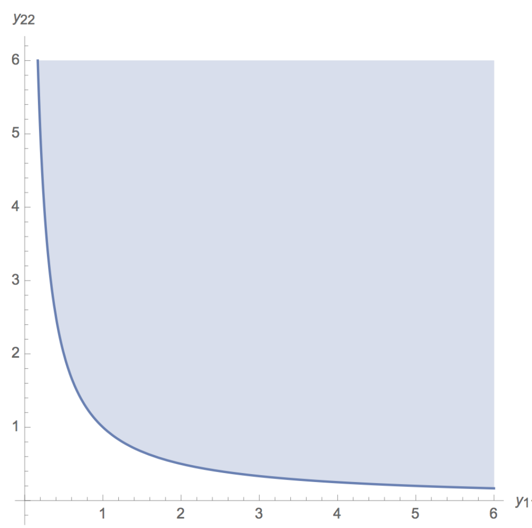
$$\inf \ y_{11}$$

$$s.t. \ y_{11}y_{22} \geq 1$$

$$y_{11} \geq 0, \ y_{22} \geq 0.$$

Highest point on degenerate parabola vs. leftmost point on hyperbola





Pathology # 2: positive duality gap

Primal:

$$\sup \; x_2$$
$$s.t. \; x_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \preceq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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Dual value is 1, and it is attained.

Bad behavior defined

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Bad behavior defined

We are curious about the semidefinite system

$$(P_{SD}) \sum_{i=1}^{m} x_i A_i \preceq B$$

- We say that it is badly behaved if $\exists c$ such that

$$\sup\{ c^T x \mid x \in (P_{SD}) \} < +\infty$$

  but the dual program has no solution with same value (i.e. dual does not attain, or positive gap).

- Well behaved, otherwise.
- A slack is $Z = B - \sum_i x_i A_i \succeq 0.$

Motivation

The systems

$$x_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \preceq \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

and

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Curious similarity – of these, and about 20 others in the literature . . . is there a combinatorial structure?

Why all bad SDPs look the same

- Semidefinite system:

$$(P_{SD}) \quad \sum_{i=1}^m x_i A_i \preceq B$$

- W.l.o.g. the max (rank) slack is

$$Z = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}.$$

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- **Then (P_{SD}) badly behaved $\Leftrightarrow$ $\exists V$ a lin. combination of the A_i as**

$$V = \begin{pmatrix} \overbrace{V_{11}}^{r} & V_{12} \\ V_{12}^T & V_{22} \end{pmatrix}, \text{ where } V_{22} \succeq 0, \ \mathrm{R}(V_{12}^T) \not\subseteq \mathrm{R}(V_{22}).$$

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- **Ex:** $x_1 \overbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}^{V} \preceq \overbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}^{Z}$

What is missing?

- Matrices Z, V prove that (P_{SD}) is badly behaved.
- But: they do not provide a "bad" c objective function
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- Aside: how do we prove that $Ax = b$ is infeasible? $\rightarrow$ row echelon form.
- We will borrow ideas from the row echelon form.

Reformulations of

$$(P_{SD}) \ \sum_{i=1}^{m} x_i A_i \preceq B$$

are obtained by a sequence of:

- Apply a rotation $V^T()V$ to all matrices, where V is invertible.
- $B \leftarrow B + \sum_{i=1}^{m} \mu_i A_i$
- $A_i \leftarrow \sum_{j=1}^{m} \lambda_j A_j$ where $\lambda_i \neq 0$
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Mostly: just elementary row operations done on (D). E.g. exchange constraints

$$A_i \bullet Y = c_i \text{ and } A_j \bullet Y = c_j$$

Theorem: (P_{SD}) is badly behaved $\Leftrightarrow$ it has a reformulation:

$$(P_{SD,bad}) \quad \sum_{i=1}^{k} x_i \begin{pmatrix} F_i & 0 \\ 0 & 0 \end{pmatrix} + \sum_{i=k+1}^{m} x_i \begin{pmatrix} F_i & G_i \\ G_i^T & H_i \end{pmatrix} \preceq \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} = Z,$$

where

1) Z is max slack; 2) $\begin{pmatrix} G_i \\ H_i \end{pmatrix}$ lin. indep. 3) $H_m \succeq 0$

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How to get there? Block Gaussian elimination!

$$\begin{pmatrix} \text{vec } A_1 \\ \vdots \\ \text{vec } A_m \end{pmatrix} \rightarrow \begin{pmatrix} \overbrace{\times}^{F_i} & \overbrace{0}^{(G_i, H_i)} \\ \times & \times \end{pmatrix}$$

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Proof that $(P_{SD,bad})$ is badly behaved:

$$x \text{ feasible} \Rightarrow x_{k+1} = \cdots = x_m = 0$$

$$\Rightarrow \quad \sup -x_m = 0$$

But: no dual soln with value 0

Example: before and after

$$\boldsymbol{x_1} \begin{pmatrix} 6 & 10 \\ 10 & 16 \end{pmatrix} \preceq \begin{pmatrix} 19 & 32 \\ 32 & 52 \end{pmatrix}$$

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After reformulation:

$$x_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \preceq \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

which is trivially badly behaved.

Large example: before reformulation

$$x_1\begin{pmatrix} 54 & 46 & 50 & 4 \\ 46 & -38 & 87 & -106 \\ 50 & 87 & -60 & 296 \\ 4 & -106 & 296 & -368 \end{pmatrix} + x_2\begin{pmatrix} 110 & 91 & 105 & -6 \\ 91 & -72 & 171 & -210 \\ 105 & 171 & -72 & 528 \\ -6 & -210 & 528 & -672 \end{pmatrix} + x_3\begin{pmatrix} 42 & 35 & 40 & 0 \\ 35 & -28 & 67 & -82 \\ 40 & 67 & -36 & 216 \\ 0 & -82 & 216 & -27 \end{pmatrix}$$

$$+x_4\begin{pmatrix} 36 & 30 & 35 & -2 \\ 30 & -24 & 57 & -70 \\ 35 & 57 & -24 & 176 \\ -2 & -70 & 176 & -224 \end{pmatrix} \preceq \begin{pmatrix} 389 & 323 & 370 & -12 \\ 323 & -257 & 610 & -748 \\ 370 & 610 & -288 & 1920 \\ -12 & -748 & 1920 & -243 \end{pmatrix}$$

Hard to tell if well or badly behaved

Large example: after reformulation

$$x_1 \left(\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) + x_2 \left(\begin{array}{cc|cc} 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) + x_3 \left(\begin{array}{cc|cc} 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & -1 \\ \hline 2 & 3 & 0 & 2 \\ 1 & -1 & 2 & 0 \end{array}\right)$$

$$+ x_4 \left(\begin{array}{cc|cc} 0 & 0 & 3 & -1 \\ 0 & 0 & 2 & -1 \\ \hline 3 & 2 & 4 & 0 \\ -1 & -1 & 0 & 0 \end{array}\right) \preceq \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

As before: $x_3 = x_4 = 0 \Rightarrow \sup -x_4 = 0$

But: no dual solution with value 0

Corollaries:

- **Similar reformulation for well-behaved systems.**
- **The question:**

  Is (P_{SD}) well behaved?

  is in $NP \cap coNP$ in real number model of computing.

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- **Similar reformulation for well-behaved systems.**
- **The question:**

  Is (P_{SD}) well behaved?

  is in $NP \cap coNP$ in real number model of computing.
- **Certificate: reformulation, and proof that Z is max rank slack.**

A "circular" proof

The bad part

(P_{SD}) **satisfies the "Bad condition"** $(\exists Z, V) \implies$

it has a "Bad reformulation" $(P_{SD,bad}) \implies$

it is badly behaved.

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Proof Linear algebra.

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The good part

(P_{SD}) satisfies the ”Good condition” $\implies$

it has a ”Good reformulation” $(P_{SD,good}) \implies$

it is well behaved.

Proof Linear algebra.

Tying it together

”Good condition” fails $\implies$ ”Bad condition” holds.

Proof Duality theorem of SDP, assuming Slater condition.

Conclusion

- **Pathologies in duality:** well- and badly behaved semidefinite (feasible) systems.
- Combinatorial type characterizations.
- **Reformulations** to easily recognize good and bad behavior $\rightarrow NP \cap co-NP$ certificates.

Papers

- **P: On the closedness of the linear image of a closed convex cone,**

  2007, Math. of OR

- **P: Bad semidefinite programs: they all look the same,**

  2010–SIOPT 2017

- **P: Characterizing bad semidefinite programs: normal forms and short proofs**

  SIAM Review, to appear

- **Others in a similar vein, with co-authors Minghui Liu, Yuzixuan Zhu, Quoc Tran-Dinh: http://gaborpataki.web.unc.edu/**

Thank you!